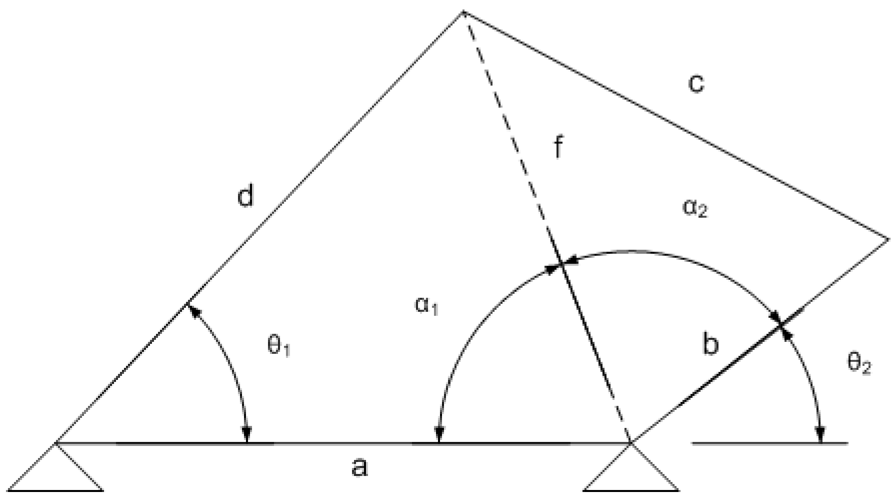
Trigonometric Solution of Four Bar Linkage

Given a 4-bar linkage with known link lengths, solve for θ_2 as a function of θ_1.



For example let $a = 3$ $b = 1.7$ $c = 2.5$ $d = 3.1$ and $\theta_1 = 45$ deg

An easy way to solve this problem is with the cosine and sine laws.

Law of cosines

$$f^2 = a^2 + d^2 - 2\cdot a\cdot d\cdot \cos(\theta_1)$$

$$d^2 = a^2 + f^2 - 2\cdot a\cdot f\cdot \cos(\alpha_1)$$

$$c^2 = b^2 + f^2 - 2\cdot b\cdot f\cdot \cos(\alpha_2)$$

Law of sines

$$\frac{\sin(\alpha_1)}{d} = \frac{\sin(\theta_1)}{f}$$

By combining the above equations, $\theta_1(\theta_2)$ can be solved for.

$$f(\theta_1) = \sqrt{a^2 + d^2 - 2\cdot a\cdot d\cdot \cos(\theta_1)}$$

$$\alpha_1(\theta_1) = \operatorname{asin}\left(\frac{d}{f(\theta_1)}\cdot \sin(\theta_1)\right)$$

or

$$\alpha_1(\theta_1) = \operatorname{acos}\left(\frac{a^2 + f(\theta_1)^2 - d^2}{2\cdot a\cdot f(\theta_1)}\right)$$

$$\alpha_2(\theta_1) = \text{acos}\left(\frac{b^2 + f(\theta_1)^2 - c^2}{2 \cdot b \cdot f(\theta_1)}\right)$$

With $\alpha 1$ and $\alpha 2$ now known, θ_2 as a function of θ_1 is then

$$\theta_2(\theta_1) = 180 \cdot \text{deg} - \alpha_1(\theta_1) - \alpha_2(\theta_1)$$

θ_2 in the example is then

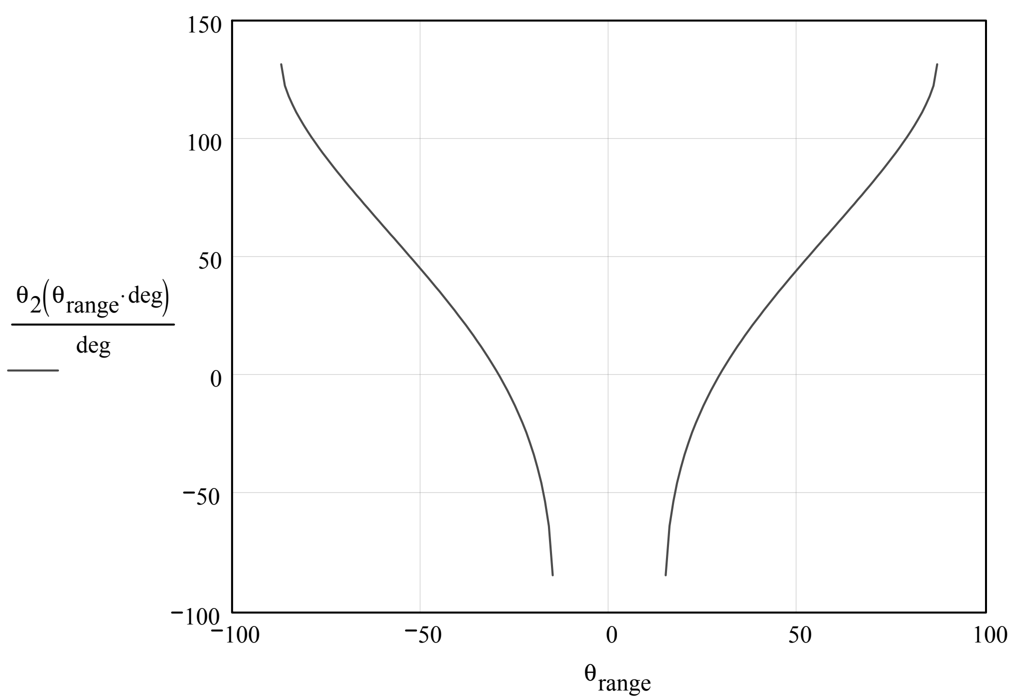
$$\theta_2(45 \cdot \text{deg}) = 35.5 \, \text{deg}$$

Further extending the example, θ_2 is plotted for a range of θ_1 values.

In this example, θ_{1min} occurs when θ_2 = -90 deg and θ_{1max} occurs when links "b" and "c" are in-line. θ_{1max} is found with the cosine law.

$$\theta_{1max} = \text{acos}\left[\frac{a^2 + d^2 - (c + b)^2}{2 \cdot a \cdot d}\right] \qquad \theta_{1max} = 87.011 \, \text{deg}$$

$$\theta_{range} = -90, -89 \, .. \, 87$$



The plot shows that these equations do not work for the special cases, i.e. $\theta_2(0) = -3.142 + 2.978i$